

## Worked Solutions

### Edexcel C4 Paper F

1.  $\frac{dy}{dt} = \cos t, \frac{dx}{dt} = 2 + \sin t$

$$\frac{dy}{dx} = \frac{\cos t}{2 + \sin t}$$

stationary points where  $\frac{dy}{dx} = 0$ .

i.e.  $\cos t = 0 \Rightarrow t = \frac{\pi}{2}, \frac{3\pi}{2}$ .

when  $t = \frac{\pi}{2}, x = 2 \cdot \frac{\pi}{2} - \cos \frac{\pi}{2} = \pi; y = 2$

$t = \frac{3\pi}{2}, x = 2 \cdot \frac{3\pi}{2} - \cos \frac{3\pi}{2} = 3\pi; y = 1 + \sin \frac{3\pi}{2} = 0$

The stationary points are  $(\pi, 2)$  and  $(3\pi, 0)$

(5)

2. (a)  $f(x) = \frac{(2+x)(3-x) - (2-x)(3+x)}{(3+x)(3-x)} = \frac{2x}{9-x^2}$

(2)

(b)  $f(x) = \frac{2x}{9\left(1 - \frac{x^2}{9}\right)} = \frac{2}{9}x \left(1 - \frac{x^2}{9}\right)^{-1}$

$$= \frac{2}{9}x \left[ 1 + (-1) \left(-\frac{x^2}{9}\right) + \frac{(-1)(-2)}{2} \left(-\frac{x^2}{9}\right)^2 + \dots \right]$$

$$= \frac{2}{9}x \left[ 1 + \frac{1}{9}x^2 + \frac{1}{81}x^4 + \dots \right]$$

$$= \frac{2}{9}x + \frac{2}{81}x^3 + \frac{2}{729}x^5 + \dots$$

(4)

3. (a)  $\frac{dm}{dt} = Ak e^{kt}$

when  $t = 0, m = 4: 4 = A e^0 \Rightarrow A = 4$

$t = 0, \frac{dm}{dt} = 8: 8 = 4k \cdot e^0 \Rightarrow k = 2$ .

(b)  $m = 4 e^{2t}$

$$20 = 4 e^{2t}$$

$$\ln 5 = 2t$$

$$t = \frac{1}{2} \ln 5 \quad (= 0.8047 \dots)$$

4.  $2x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$

$$\frac{dy}{dx}(x + 2y) = -(2x + y)$$

$$\frac{dy}{dx} = -\frac{(2x + y)}{x + 2y}$$

at  $(1, 2)$  gradient  $= -\frac{4}{5}$

equation of tangent is  $y - 2 = -\frac{4}{5}(x - 1)$

$$5y + 4x = 14$$

5. (a) area  $= \int_0^4 (2x + 1)^{\frac{1}{2}} dx = \left[ \frac{2}{3} \times \frac{1}{2} (2x + 1)^{\frac{3}{2}} \right]_0^4 = \frac{1}{3} \cdot 9^{\frac{3}{2}} - \frac{1}{3} \cdot 1^{\frac{3}{2}} = \frac{1}{3} (27 - 1) = \frac{26}{3}$

(b) volume  $= \pi \int_0^4 (2x + 1) dx = \pi \left[ x^2 + x \right]_0^4 = 20\pi$  units<sup>3</sup>

6. (a) At point of intersection  $2 + \lambda = -3 + 2\mu$

$$3 + 4\lambda = 4 + \mu$$

$$-1 + 2\lambda = -2 + \mu$$

Solving first two equations,  $\lambda = 1$  and  $\mu = 3$ .

The point of intersection is at  $(3, 7, 1)$ .

(b) let angle between  $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$  and  $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$  be  $\theta$ .

$$\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \sqrt{21} \times \sqrt{6} \cos \theta$$

$$2 + 4 + 2 = \sqrt{21}\sqrt{6} \cos \theta$$

$$\theta = 44.54 \dots = 45^\circ, \text{ to the nearest degree}$$

(c) line meets the  $yz$ -plane at  $x = 0$ .

$$\therefore 2 + \lambda = 0, \quad \lambda = -2$$

line meets plane at  $\begin{pmatrix} 0 \\ -5 \\ -5 \end{pmatrix}$

$$\begin{aligned} 7. (a) \int_a^{a+h} (x^2 - a^2) dx &= \left[ \frac{1}{3}x^3 - a^2x \right]_a^{a+h} \\ &= \frac{1}{3}(a+h)^3 - a^2(a+h) - \left( \frac{1}{3}a^3 - a^3 \right) \\ &= \frac{1}{3}(a^3 + 3a^2h + 3ah^2 + h^3) - a^3 + a^3 \\ &= \frac{1}{3}h^2(a+h) \end{aligned}$$

$$(b) \int \tan^2 x \, dx = \int (\sec^2 x - 1) dx = \tan x - x + c$$

$$\begin{aligned} (c) \int_0^{\frac{\pi}{3}} x \sec^2 x \, dx &= \int_0^{\frac{\pi}{3}} x \frac{d}{dx}(\tan x) dx \\ &= \left[ x \tan x \right]_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} \tan x \, dx \\ &= \left[ x \tan x + \ln \cos x \right]_0^{\frac{\pi}{3}} \\ &= \frac{\pi}{3} \sqrt{3} + \ln \frac{1}{2} - (0 + \ln 1) = \frac{\pi}{3} \sqrt{3} - \ln 2 \end{aligned}$$

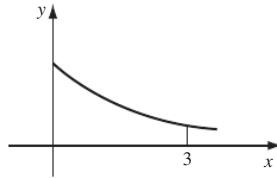
8. (a)

$x$	0	1	2	3
$\frac{4}{x+3}$	$\frac{4}{3}$	1	$\frac{4}{5}$	$\frac{2}{3}$

$$\text{integral} = \frac{1}{2} \left[ \frac{4}{3} + \frac{2}{3} + 2 \left( 1 + \frac{4}{5} \right) \right] = \frac{14}{5} \quad (4)$$

$$\begin{aligned} (b) \int_0^3 \frac{4}{x+3} dx &= \left[ 4 \ln(x+3) \right]_0^3 \\ &= 4 \ln 6 - 4 \ln 3 \\ &= 4 \ln 2 \\ &= \ln 16 \end{aligned} \quad (3)$$

(c)



sum of trapezia > actual area under curve.

$$\frac{14}{5} > \ln 16 \quad (3)$$

$$\begin{aligned} 9. (a) \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x} dx &= \left[ \ln \sin x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \\ &= \ln 1 - \ln \frac{1}{2} \\ &= \ln 2 \end{aligned}$$

$$(b) \int \frac{1}{y} dy = \int \left( \frac{1-x}{1+x} \right) dx$$

By division  $\frac{-1}{x+1} - \frac{x+1}{x+1}$

$$\int \frac{1}{y} dy = \int \left( -1 + \frac{2}{1+x} \right) dx$$

$$\ln y = -x + 2 \ln(1+x) + c$$

$$y = 4, \quad x = 0 :$$

$$\ln 4 = 2 \ln 1 + c$$

$$c = \ln 4$$

$$\ln y = -x + 2 \ln(1+x) + \ln 4$$

$$\ln y = \ln 4(1+x)^2 - x$$